

# A New Method for the Design of a Quasi-Optical Mode Converter With a Special Reflector

Shiwen Yang, Soon Hie Tan, and Hongfu Li

**Abstract**—This paper presents a new method for the design of a special reflector for quasi-optical mode converters, using the combination of vector diffraction theory and geometrical optics. The reflector is formed in a piecewise manner, according to a base reflector shape chosen previously. Each piece of reflector cell is adjusted separately. A numerical example is carried out for the quasi-optical mode conversion of a circular waveguide  $TE_{52}$  mode at 35 GHz, and the calculated results show that the output Gaussian wave beam waist radius is only less than twice the wavelength.

**Index Terms**—Mode conversion, quasi-optics.

## I. INTRODUCTION

THE application of high-power microwave (HPM) sources, such as in gyrotron oscillators and gyrotron amplifiers, usually requires that the HPM be linearly polarized and well defined in shape, like the Gaussian wave beam propagating in free space or the  $HE_{11}$  mode in a corrugated waveguide. However, the output modes of currently available HPM sources are generally circular waveguide modes, or whispering gallery modes (WGMs). Therefore, it is absolutely necessary to transform these circular waveguide modes into  $TE_{11}$  mode or  $HE_{11}$  mode in the corrugated waveguide, or the Gaussian wave beam in free space. One of the mode transformation methods is the adoption of a waveguide mode converter, which proves to be simple and efficient for lower order modes [1]. However, for higher order mode or WGM, the use of waveguide mode converters is complicated and inefficient.

Another alternative technique is to use the quasi-optical mode converter system, which was proposed by Vlasov *et al.*, to transform the circular waveguide modes into a Gaussian wave beam in free space [2]. In general, it consists of a waveguide opening, known as the Vlasov launcher, and a series of reflectors. Since then, much effort has been given to improve its characteristics, including the deformation in the feeding waveguide and the design of the reflector system. For the launcher analysis, the commonly used methods include geometric optics (GO) [3], [4], scalar diffraction theory [5], [6], and vector diffraction theory [7], [8], while for the reflector shape design, only GO [3], [4] and scalar diffraction theory were mostly used [5], [6]. In [8], the

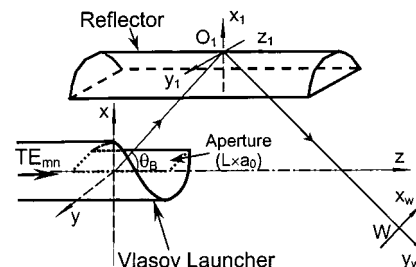


Fig. 1. Quasi-optical mode converter system.

launcher is analyzed using the vector diffraction theory, however, the reflector shape was determined using sheer GO approximation, by assuming the radiation source to be a part of the caustic surface at the launcher. Thus, the output wave beam waist radius is generally greater than three times the wavelength. For lower frequency gyrotrons, this will require a corrugated waveguide with relatively larger diameter to receive the output Gaussian beam for further application.

Based on the combination of vector diffraction theory and GO, this paper proposes a new method for the design of the quasi-optical reflector system. The reflector is formed in a piecewise manner, based on a base reflector chosen previously. First, the position of each reflector cell was determined by GO in phase condition. Then, the normal direction of each reflector cell, which was usually not taken into account by previous researchers, was determined by GO reflective law. Also, both the incident wave phase and wave vector were directly obtained from the vector diffracted field of the launcher, not from the assumption of a caustic surface or the geometric rays of the launcher. Using this method, it is possible to adopt the conventional Vlasov launcher and only one reflector is required to transform the circular waveguide mode into a very narrow Gaussian wave beam, with a beam waist radius of less than twice the wavelength. Numerical results are presented for the quasi-optical mode conversion of a circular  $TE_{52}$  output mode at 35 GHz.

## II. THEORY

The whole scheme of the quasi-optical mode converter is shown in Fig. 1. Suppose a right-hand rotating circular waveguide mode  $TE_{mn}$  is propagating in a circular waveguide with radius  $a_0$ . To unwind the  $TE_{mn}$  mode into a linearly polarized free-space Gaussian mode, the conventional helical Vlasov launcher is employed to launch the rotating WGM into free space in a moderate wave beam. The appropriate length of the straight cut of the launcher was proposed by Mübius *et al.*, by

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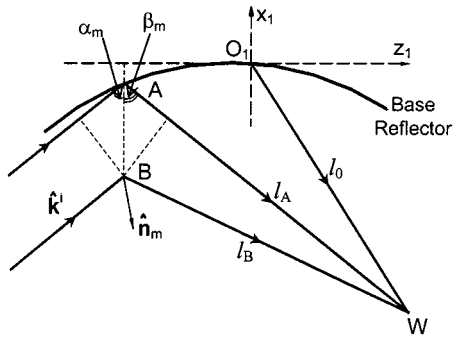


Fig. 2. Schematic drawing for the design of the special reflector.

matching the normal power flux integral over both the waveguide cross section and the rectangular aperture of width  $a_0$  and length  $L$  in the  $y$ - $z$ -plane [7]

$$L = L_B \frac{\mu_{mn}^2 - m^2}{\mu_{mn}} \frac{J_m^2(\mu_{mn})}{1 - J_0^2(\mu_{mn}) + J_m^2(\mu_{mn}) - 2 \sum_{k=1}^m J_k^2(\mu_{mn})} \quad (1)$$

where  $\mu_{mn}$  is the  $n$ th nonzero root of Bessel function  $J'_m(x)$ ,  $L_B = 2\pi a_0 / tg\theta_B$  is the bounce length,  $\theta_B = \arccos(\beta/k_0)$  is the bounce angle,  $k_0$  and  $\beta$  is the propagating constant in free space and waveguide, respectively.

The radiation fields of the launcher are supposed to be the radiation of the unperturbed waveguide fields on a rectangular aperture, which is bounded by the waveguide axis and the surface (in the axial direction), and the radial boundaries at the origin and end of the helical cut (in the transverse direction). Since the aperture size is not very large compared to the wavelength, it is necessary to use the vector diffraction theory to calculate the radiation field. The Stratton–Chu formula can be used to calculate the vector diffraction fields  $\mathbf{E}'$  and  $\mathbf{H}'$  at the observing point  $P(x', y', z')$

$$\mathbf{E}' = \iint_S \{ -j\omega\mu(\mathbf{n} \times \mathbf{H})G + (\mathbf{n} \times \mathbf{E}) \times \nabla G + (\mathbf{n} \cdot \mathbf{E})\nabla G \} dS \quad (2)$$

$$\mathbf{H}' = \iint_S \{ j\omega\epsilon(\mathbf{n} \times \mathbf{E})G + (\mathbf{n} \times \mathbf{H}) \times \nabla G + (\mathbf{n} \cdot \mathbf{H})\nabla G \} dS \quad (3)$$

where  $G$  is the Green function for a point source in the free space

$$G = \frac{e^{-jk_0 R}}{4\pi R} \quad (4)$$

and  $R$  is the distance between the source point  $Q(x, y, z)$  and the observing point  $P(x', y', z')$ .

The procedure for the design of the special reflector begins with Fig. 2. Two objectives are to be satisfied in the reflector design. Firstly, the phase of the reflected wave should be constant at the output point. Secondly, all the incident waves should reach the same output point  $W$  after being reflected by the reflector.

In the first step of the design procedure, a base reflector with a regular shape is chosen. Possible reflectors are a part of a spherical surface, a cylindrical surface, or a flat mirror. The projection of the base reflector onto the  $y_1$ - $z_1$ -plane is a rectangular region with the dimension of  $W_y \times W_z$ . The rectangular projection region is discretized into  $n_y \times n_z = N$  small rectangular cells, and the base reflector is also discretized into  $N$  small curved surface cells. If  $N$  is large enough, each small curved surface cell can be replaced by a small rectangular plane cell, with the same projection cell size on the  $y_1$ - $z_1$ -plane. According to the GO approximation, the incident field at the  $m$ th curved surface cell, denoted as point  $A$ , can be approximated as plane waves, whose unit wave vector  $\hat{\mathbf{k}}^i$  can be determined by the incident field  $\mathbf{E}^i$  and  $\mathbf{H}^i$ . Since the waves radiated from the Vlasov launcher are still the interferential waves, the phase differences among each field components are still constant. Therefore, the phase of an arbitrary component, e.g.,  $E_y$ , can be chosen to represent the phase  $\varphi^i$  of incident plane waves at  $A$ . If the reflector cell at  $A$  cannot satisfy the phase constant condition at  $W$ , that reflector cell can be moved along the  $-x$  axis, with a small deviation  $\Delta_k$  to point  $B$ , to satisfy the phase constant condition. By choosing the origin  $O_1$  of the  $x_1$ - $y_1$ - $z_1$  coordinate system as the reference point, the phase constant condition at  $W$  can be expressed as

$$\varphi^i(B) - \pi - k_0 l_B = \text{constant} \equiv \varphi^i(O_1) - \pi - k_0 l_0. \quad (5)$$

For a small  $\Delta_m$ , the incident wave vectors at point  $A$  and  $B$  can be approximated as having the same directions. Hence,

$$\varphi^i(B) \approx \varphi^i(A) + k_0 \Delta_m \cos \alpha_m \quad (6)$$

$$l_B \approx l_A - \Delta_m \cos \beta_m \quad (7)$$

where  $\alpha_m$  is the angle between the wave vector at the  $m$ th cell and the  $x$  axis,  $\beta_m$  is the angle between  $AW$  and  $x$  axis,  $l_0$ , and  $l_A$  and  $l_B$  are the lengths of the line segments  $O_1W$ ,  $AW$  and  $BW$ , respectively. From (5)–(7),  $\Delta_m$  is given by

$$\Delta_m = \frac{\varphi^i(O_1) - \varphi^i(A) + k_0(l_A - l_0)}{k_0(\cos \alpha_m + \cos \beta_m)}. \quad (8)$$

Secondly, in order to make the plane waves, reflected by each small rectangular plane cells at  $B$ , reach the same output point  $W$ , it is necessary to adjust the normal unit vectors of each small rectangular plane cell  $\hat{\mathbf{n}}_m$  to satisfy the Snell's reflection law, namely,

$$\hat{\mathbf{k}}^i \cdot \hat{\mathbf{n}}_m = -\hat{\mathbf{n}}_m \cdot \hat{\mathbf{l}}_B \quad (9)$$

where  $\hat{\mathbf{k}}^i$  is the unit vector of the incident wave vector,  $\hat{\mathbf{l}}_B$  is the unit vector of  $BW$ , respectively, and  $\hat{\mathbf{k}}^i$ ,  $\hat{\mathbf{l}}_B$ ,  $\hat{\mathbf{n}}_m$  all are in a same plane.

Now, the equation of the  $m$ th small rectangular plane cell at  $B$  can be determined by  $\hat{\mathbf{n}}_m$  and  $B$ 's coordinates, namely,  $(x_{1A} - \Delta_m, y_{1A}, z_{1A})$ . The whole special reflector can be determined by the  $N$  cells of such small rectangular plane reflectors, as long as  $N$  is large enough. After the shape of the special reflector is

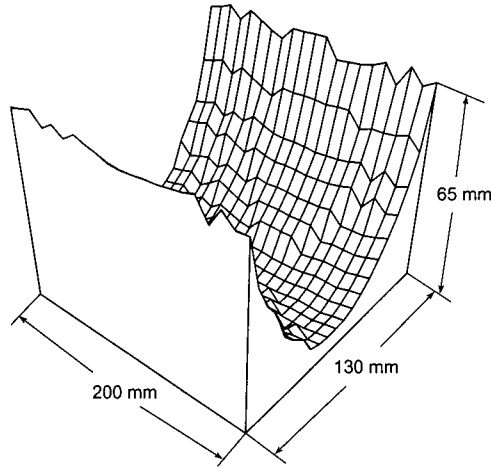


Fig. 3. 3-D shape of the special reflector for the  $TE_{52}$  mode conversion.

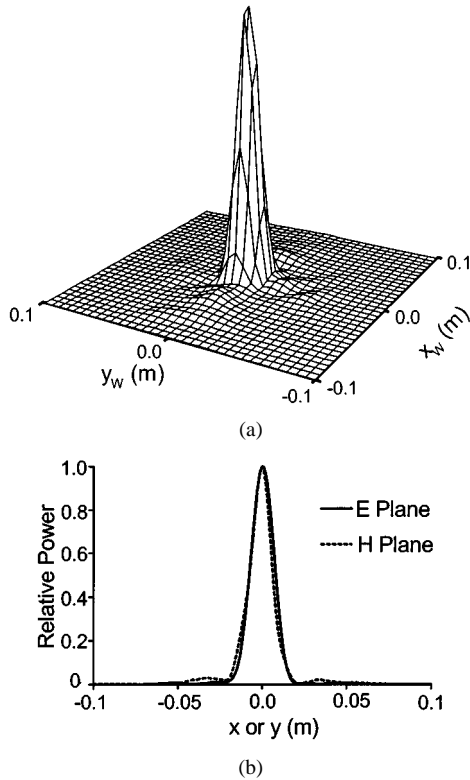


Fig. 4. Output wave beam profile. (a) 3-D relative power distribution. (b)  $E$ - and  $H$ -plane relative power distribution.

determined, the actual magnetic field distribution at the output point  $W$  can be calculated using the vector diffraction integral

$$\mathbf{H}_W = \int_S 2(\hat{\mathbf{n}} \times \mathbf{H}^i \times \nabla G) dS. \quad (10)$$

### III. NUMERICAL EXAMPLES

The above synthesis procedure is applied to the design of a quasi-optical mode converter to transform the circular waveguide  $TE_{52}$  mode, propagating in a 36.0-mm-diameter waveguide at  $f_0 = 35$  GHz, into a Gaussian-like wave beam with small beam waist radius. The converter scheme is shown in

Fig. 1. The length of the Vlasov launcher's straight cut, according to (1), is equal to 70.0 mm. The bounce angle of the launching beam is about  $52.8^\circ$ . The output point  $W$  is located at 250 mm away from the reference point  $O_1$  in the reflected line, forming an angle of  $\theta_B$  with respect to the  $z$  axis. The base reflector is chosen to be a part of a cylindrical surface with a radius of 70 mm. The dimensions of the reflector are about 200 mm  $\times$  130 mm  $\times$  65 mm. The three-dimensional (3-D) shape of the special reflector is shown in Fig. 3.

Fig. 4(a) shows the 3-D relative power distribution on the output plane, defined in the transverse direction of the output wave beam at the output point  $W$ . The calculation shows that the power propagation efficiency on the output plane is about 99.4%, while about 90% of the output power on the plane is concentrated within the beam waist. Further higher power conversion efficiency could be expected by optimizing the geometrical parameters. Fig. 4(b) shows the relative power distribution in the  $E$ - and  $H$ -planes at  $W$ . The power density falls to  $1/e^2$  of its on-axis value, at about 13 mm in the  $y$  direction ( $E$ -plane) and 14 mm in the  $x$  direction ( $H$ -plane). Thus, the output beam waist radii are only within about twice the wavelength, which are quite smaller than those of the previous results [6], [8]. The small side lobes especially in the  $H$ -plane are probably due to the phase approximation at point  $B$ , as shown in (6).

### IV. CONCLUSION

A new method which combines vector diffraction theory and GO has been presented for the design of a quasi-optical mode converter with a special reflector. The special reflector is formed by adjusting the position and the unit normal vector's direction of each small reflector cell, according to the requirements of GO, while the incident wave vectors and the incident and reflected fields are calculated using the more accurate vector diffraction theory. A numerical example is presented for the transformation of the  $TE_{52}$  mode into a Gaussian-like wave beam at 35 GHz. The calculated results show that the output wave beam waist radii is 14 mm in the  $x$  direction and 13 mm in the  $y$  direction, which are only less than twice the wavelength.

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